Expectations and volatility

Empirical patterns and behavioral heterogeneity

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Abstract

This paper investigates the relationship between expectations and volatility. Data on empirical expectations show a strong comovement between forecasts and economic variables. The causality seems to run in both directions, however. As recent experimental research on heterogeneous expectations has shown that agents use different forecasting heuristics such as adaptive, trend-following and anchoring rules, these expectation measures can be constructed based on aggregate time series. A structural estimation based on macroeconomic data for the European economy between 1995 and 2017 provides evidence for large shares of trend-following heuristics. Allowing for time-varying shares yields a situation close to perfect heterogeneity. Overall, the share of trend-followers seems to increase in booms and fall in recessions. Adaptive expectations follow the opposite pattern.

Keywords: heterogeneous expectations, evolutionary selection, DSGE models

JEL Classification: C52, D84, E32

1 Introduction

This paper investigates the relations between expectations and volatility. Agents' decisions related to investment and consumption are influenced by their expectations. Hence, the evolution of economic variables depends on expectations and the volatility of expectations should also affect the volatility of economic variables, i.e.

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$\Delta E_t \Rightarrow \Delta Y_t$. Under rational expectations, agents have perfect information about the economy and volatility arises only through exogenous shocks. As empirical expectations often deviate from model-consistent expectations, their contribution to aggregate fluctuations can be examined. High volatility, however, is detrimental to long-run growth. The contribution of the present paper is twofold: First, only the link between expectations and volatility is examined, while the relation between volatility and growth has already been studied extensively in the literature. Furthermore, the paper estimates the degree of heterogeneity in the economy for European data. Previous research has already dealt with related models with at least two types of forecasts (see Hommes 2013, p.196ff and the references therein). Boswijk/Hommes/Manzan (2007) analyze stock prices and explain bubbles and crashes by time-varying fractions of fundamental and trend-following expectations. Cornea/Hommes/Massaro (2012) estimate a Phillips curve and find a correct positive sign for output gap, once they consider nonrational expectations with two types. Cole/Milani (2016) estimate a DSGE-VAR model, where rational expectations are replaced by boundedly rational expectations as well as by heterogeneous expectations. They also find significant shares of expectation shocks that would be zero in rational expectations models. Section 2 looks at the correlation patterns of empirical expectations and economic variables such as output gap, consumption and investment. Section 3 estimates the degree of heterogeneity with fixed and time-varying group shares. Section 4 concludes.

2 Empirical expectations and volatility

In general, empirical expectations of agents and experts are measured by qualitative questions and quantified by calculating balance scores as the difference of positive and negative replies (see Curtin 2007, p.7/p.14). Data on empirical expectations are available from surveys that are regularly conducted by the Directorate General for Economic and Financial Affairs (see European Commission 2017). The Economic Sentiment Indicator (ESI) for the European Union is based on five sectors: Business, Services, Consumers, retail, construction. “About 137 000 firms and more than 41 000 consumers are currently surveyed every month across the EU.” (European Commission 2017, p.4) The ESI is aggregated across countries and sectors and seasonally adjusted.

As a matter of robustness, expectations from the German ifo institute are also considered. The ifo Economic climate for the Eurozone and its components, economic situation and expectations for the next six months, are part of the World Economic

	mean	median	standard deviation	skewness	kurtosis	stationarity
ESI	101.6796	103.65	9.15382	-1.158779	5.006377	0.0026
y	0.0000	-0.001138	0.0112017	0.4422734	4.343126	0.0006
c	0.0000	-0.001101	0.0067674	0.6882909	3.204664	0.0018
inv	0.0000	0.000694	0.0279071	0.6812327	3.777563	0.0026

Table 1: Descriptive Statistics

Survey and are available since 1990. These measures of expectations avoid indexation and are based on the balance of positive and negative expert forecasts (see Garnitz/Wollmershäuser 2017).

Economic variables such as output and its components such as consumption of private households and investment (gross fixed capital formation) are available from Eurostat. Output is measured by quarterly, seasonally adjusted real GDP. The same standards are applied to consumption and investment. To get the cyclical variation of these variables, the data are HP filtered with a factor $\lambda = 1600$, as is standard in the literature. The period from 1995:Q1 to 2017:Q2 is considered, which yields 90 observations for each variable.

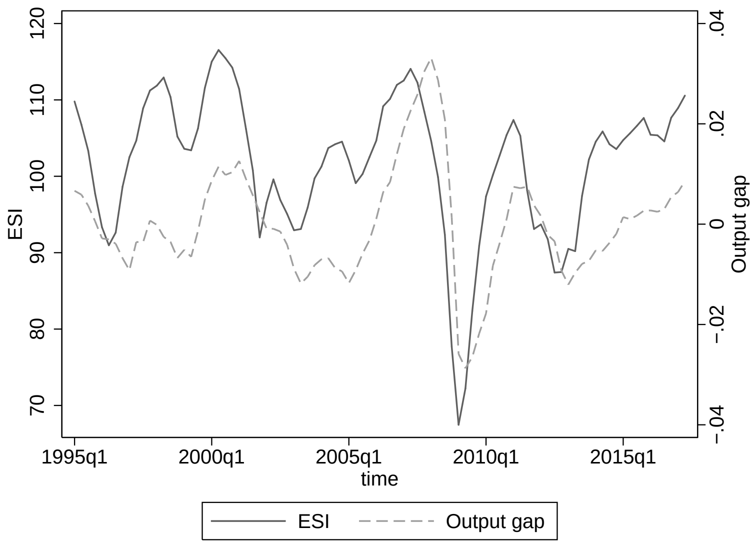

Figure 1: Time paths of expectations and output gap

Figure 1 shows a strong comovement of expectations and output gap. Moreover, a close inspection of the time series suggests that a change in economic sentiments precedes a change in output, which might be related to waves of optimism and pessimism that drive cyclical output. Figure 2 shows the time series of ifo climate and expectations. As both measures refer to the Eurozone, the output gap is calculated accordingly.

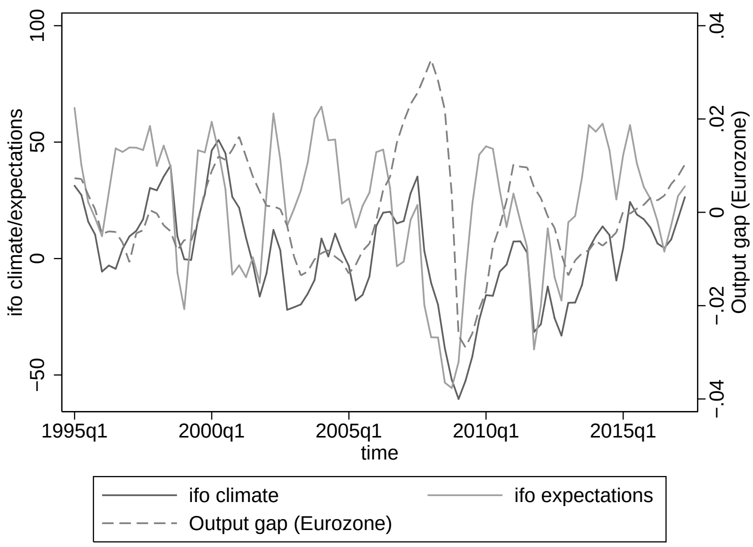

Figure 2: Time paths of ifo climate/expectations and output gap

	mean	median	standard deviation	skewness	kurtosis	stationary
ifo climate	2.226667	4.3	22.29865	-0.3899526	3.171409	0.0203
ifo situation	-14.06444	-18.1	34.27762	0.1871057	2.576927	0.0091
ifo expectations	22.86333	27.4	28.14743	-0.8112109	3.149966	0.0035
y	0.0000	-0.0005622	0.0117006	0.3802435	3.652661	0.0005
c	0.0000	-0.0004111	0.0071032	0.1190862	2.04757	0.0044
inv	0.0000	-0.0034608	0.0268246	0.6285204	3.114555	0.0011

Table 2: Descriptive Statistics II

Table 1 reports descriptive statistics of economic sentiments and cyclical variables for the European Union. Economic sentiments are skewed to the left, while the other variables are skewed to the right. All variables exhibit excess kurtosis greater than 3. An augmented Dickey-Fuller test is applied to test the presence of a unit root. All time series are stationary.

On average, ifo climate and ifo expectations are positive, while the current situation is negative and has the highest standard deviation (see Table 2). Climate and expectations are skewed to the left, while all other variables are skewed to the right. With the exception of consumption and the current situation, all variables have a slight excess kurtosis.

Table 3 reports the autocorrelations together with the correlations between economic sentiments and output gap, consumption and investment. First, all variables exhibit a very high persistence around 0.9 with the exception of ifo expectations and ifo climate. Second, all variables are positively correlated with economic sentiments. The correlations are higher for the ESI than for the ifo data. Third, the current economic situation is even stronger correlated with current economic variables. And

autocorrelations		correlations		
ESI	0.8938	ESI		
y	0.9032	0.5811		
c	0.9026	0.4784		
inv	0.8996	0.4868		
ifo climate	0.843			
ifo situation	0.9075			
ifo expectations	0.7663	ifo climate	ifo situation	ifo expectations
y	0.8973	0.4544	0.7414	-0.2791
c	0.8966	0.3295	0.6407	-0.3395
inv	0.8963	0.3644	0.7322	-0.4151

Table 3: Autocorrelations and correlations

finally, ifo expectations for the next six months are negatively correlated with all economic variables. The ifo data allow to decompose economic sentiments into a current and a future component. Overall expectations move in the same direction as current economic variables, but they are not exclusively influenced by the current situation. The ESI and ifo climate are strongly positively correlated with a correlation coefficient of 0.9083, which implies that both measures capture the same underlying sentiments. However, the correlation between ESI and the current situation is 0.8030, while the correlation with ifo expectations is only 0.4056. This shows that the ESI index is much more correlated with the current ifo climate than with the ifo prospects for the next 6 months.

Crosscorrelations up to five leads and lags between expectations and cyclical variables are shown in Figure 3 and 4. Cyclical output as well as its components, consumption and investment, show the same correlation structure with economic sentiments. Present and future values are positively correlated, while past values are negatively correlated.

Finally, a Granger causality test based on a VAR model with five lags is conducted to test whether expectations drive cyclical output, as Figure 1 and 2 suggest, or whether they can be predicted by past realizations of output.

According to the first part of Table 4, the ESI index influences output gap, but it is also influenced by past changes in output gap. The causality seems to run in both directions. This confirms a similar result by Curtin (2007, p.14). A similar test based on the ifo data yields slightly different results. The null hypothesis that ifo climate does not Granger-cause output gap cannot be rejected. However, output gap helps to predict ifo climate. Hence, economic variables are indeed influenced by expectations, but expectations themselves are also endogenous with respect to the evolution of the economy. This result casts doubt on rational expectations and

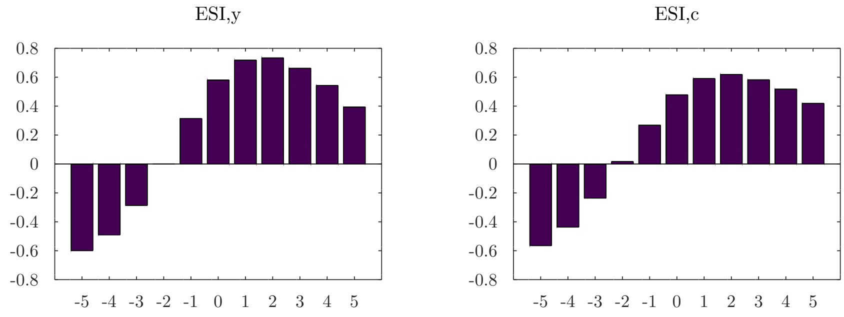

Figure 3: Crosscorrelations with sentiments, own figure

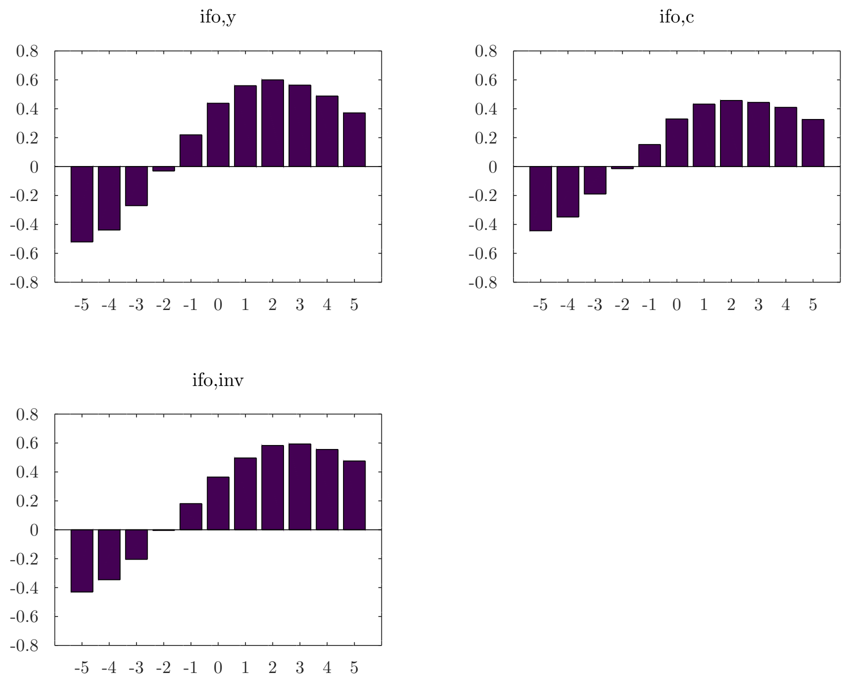

Figure 4: Crosscorrelations with ifo climate, own figure

Equation	Excluded	chi2	df	Prob>chi2
y	ESI	14.447	5	0.013
y	ALL	14.447	5	0.013
ESI	y	21.419	5	0.001
ESI	ALL	21.419	5	0.001

Equation	Excluded	chi2	df	Prob>chi2
y	ifo climate	8.2331	5	0.144
y	ALL	8.2331	5	0.144
ifo climate	y	21.848	5	0.001
ifo climate	ALL	21.848	5	0.001

Table 4: Granger causality tests

supports the idea of bounded rationality as well as the view that markets can be characterized as "expectations feedback systems" (see Hommes 2013, p.6). The next step is therefore to impose more structure on the data, i.e. to specify deductively the way according to which expectations are formed and to estimate the degree of heterogeneity that is consistent with aggregate fluctuations.

3 Estimating the degree of heterogeneity in the economy

To estimate the degree of heterogeneity, measures of heterogeneous expectations are constructed.

3.1 Construction of expectations

As we know from experimental research, expectations are heterogeneous and boundedly rational, instead of homogeneous and rational. The work of Anufriev/Hommes (2012) and Assenza et al. (2014) has shown that agents form their forecasts based on four strategies, which explain asset-price bubbles and financial dynamics. In line with these experimental results, I construct expectation measures based on empirical data for adaptive expectations, weak and strong trend-followers as well as for the anchoring rule.

$$ADA: x^e_{1,t+1} = 0.65x_{t-1} + 0.35x^e_{1,t} \tag{3.1}$$

$$WTR: x^e_{2,t+1} = x_{t-1} + 0.4(x_{t-1} - x_{t-2}) \tag{3.2}$$

$$STR: x^e_{3,t+1} = x_{t-1} + 1.3(x_{t-1} - x_{t-2}) \tag{3.3}$$

$$LAA: x^e_{4,t+1} = 0.5(x^{av}_{t-1} + x_{t-1}) + (x_{t-1} - x_{t-2}) \tag{3.4}$$

The first rule represents adaptive expectations, while the three remaining rules are trend-following strategies. Here, the calculation of the average in the last heuristic is only based on the last 8 observations as in Cole/Milani (2016, p.13).

3.2 Structural Estimation

At first, fixed shares of the expectation rules are structurally estimated using Bayesian methods. Estimation is based on the canonical New Keynesian model with three equations:

$$y_t = y^e_{t+1} - \phi(i_t - \bar{\pi}^e_{t+1}) + g_t \tag{3.5}$$

$$\pi_t = \lambda y_t + \rho \bar{\pi}^e_{t+1} + u_t \tag{3.6}$$

$$i_t = \bar{\pi} + \Phi_\pi(\pi_t - \bar{\pi}) + k_t \tag{3.7}$$

The calibration follows Clarida/Galí/Gertler (2000, p.170) and assumes $\phi = 1$, $\lambda = 0.3$ and $\rho = 0.99$. The inflation target is $\bar{\pi} = 2$ and the reaction coefficient of monetary policy is set to $\phi_\pi = 1.5$. Shock variances have a standard error of 1 and an autocorrelation of 0.5. Rational expectations for the output gap are replaced by a weighted average of adaptive expectations, trend-followers and anchoring expectations:

$$y^e_{t+1} = n_1 * y^e_{1,t+1} + n_2 * y^e_{2,t+1} + n_3 * y^e_{3,t+1} + (1 - n_1 - n_2 - n_3) * y^e_{4,t+1} + e_t \tag{3.8}$$

The expectation shock e_t is added to account for expectational dynamics that cannot be captured by heterogeneous expectations. The prior weights for the four heuristics are all assumed to be 0.25 and to follow a Beta distribution. As inflation rates are not observed and the Blanchard-Kahn conditions have to be satisfied, the assumption of rational expectations is maintained for this variable. Estimation is based on the Metropolis-Hastings algorithm with 20.000 replications and a burn-in of 2000. Only output gap is observed. In a first step, the DSGE model is estimated. The results

	prior mean	posterior mean	90% HPD interval
DSGE			
n_1	0.25	0.2151	0.0700 0.3546
n_2	0.25	0.3196	0.1282 0.5045
n_3	0.25	0.4628	0.2496 0.6743
DSGE-VAR			
n_1	0.25	0.2241	0.1097 0.3085
n_2	0.25	0.1836	0.0545 0.2993
n_3	0.25	0.3398	0.2484 0.4197
DSGE prior weight	1.000	0.0692	0.0667 0.0727

Table 5: Parameter estimates

are shown in Table 5. The posterior fractions deviate substantially from the priors. Strong trend-followers have the highest share with 46.28%. Weak-trend followers have a share of 31.96 %, while adaptive expectations, which are most closely related to fundamentalists, have a share of slightly more than one fifth. The anchoring rule represents the most sophisticated rule, but it is used by only 0.25% of agents.

Now, a DSGE-VAR model can be estimated in order to let the data decide whether the DSGE restrictions are binding. If the hyperparameter λ is zero, the restrictions should be dropped in favor of the VAR model, while a hyperparameter $\lambda = \infty$ indicates that the restrictions are fulfilled. The DSGE prior weight indicates that the DSGE model is heavily misspecified, so that the DSGE-VAR model puts now more weight on the atheoretical VAR than on the DSGE model. This time the share of adaptive expectations is only slightly higher, but weak and strong trend-followers have now a much lower share. The share of weak trend-followers is lower than one fifth, while strong trend-followers have a share of one third. The learning anchoring and adjustment rule is now hundred times more important with a share of one quarter, ie. 25.25%. Cole/Milani (2016) assumed heterogeneous expectations for both output gap and inflation. However, rational expectations are also part of their concept of heterogeneous expectations and they do not differentiate between weak and strong trend-followers. They find a small share of the anchoring rule, while adaptive expectations are most important. Their results are thus at odds with the findings here, where trend-followers have the highest share and the anchoring rule is more important than adaptive expectations.

A Forecast error variance decomposition based on the estimated models decomposes volatility into structural shocks components and an expectations-related component and yields relatively similar results (see Table 6). Expectation shocks account for substantial shares of overall variance, especially for nominal interest rates. Output gap is mainly driven by inflation shocks, inflation is driven by demand, interest

	ϵ_g	ϵ_u	ϵ_k	ϵ_e
DSGE				
y	14.45	56.65	14.45	14.45
y^e	11.97	46.95	11.97	29.10
π	29.24	12.28	29.24	29.24
i	38.21	16.01	7.57	38.21
DSGE-VAR				
y	14.45	56.65	14.45	14.45
y^e	11.63	45.61	11.63	31.13
π	29.57	11.29	29.57	29.57
i	39.28	14.99	6.44	39.28

Table 6: Forecast error variance decomposition

and expectation shocks and interest rates are driven by demand shocks and expectation shocks. Overall, the expectation shocks are smaller than those reported in Cole/Milani (2016).

3.3 Time-varying fractions

The next step concerns time-varying fractions as well as the estimation of the learning speed. This helps to clarify whether the variation in expectations comes from forecasts that are adjusted to different circumstances or from shifting group shares. Furthermore, there is also experimental evidence for evolutionary switching between different heuristics (see Pfajfar/Žakelj 2014). With time-varing fractions, utility depends on *past* performance. In contrast to Cornea/Hommes/Massaro (2012), four instead of two groups are assumed. Moreover, output gap rather than inflation is considered. This estimation does not require a structural model, however, the switching mechanism according to which agents react to differences in forecasting errors imposes some structure on the data. The forecast error is the absolute value of the difference between the last expectation and the realized variable (see Cornea/Hommes/Massaro 2012, p.15):

$$FE_t^i = |y_{t-1}^e - y_t| \tag{3.9}$$

Utility depends negatively on the forecast error of a particular heuristic relative to all forecast errors and the time-varying group share $n_{i,t}$ depends on this fitness measure:

$$U_{i,t} = -\frac{FE_t^i}{\sum_{i=1}^{I} FE_t^i} \tag{3.10}$$

	EU	Eurozone
β	2.5675	3.4740
	(0.145)	(0.100)
Observations	80	80
R^2	0.9015	0.8879

p-values in parentheses
* $p < 0.10$, ** $p < 0.05$

Table 7: NLS estimation

$$n_{i,t} = \frac{exp(\beta U_{i,t-1})}{\sum_{i=1}^{I} exp(\beta U_{i,t-1})} \tag{3.11}$$

Variations in output gap are assumed to be entirely driven by the four heuristics and endogenous switching between these strategies:

$$\begin{aligned} y_t = &\frac{exp(\beta * U_{1,t-1})}{\sum_{i=1}^{I} exp(\beta U_{i,t-1})} * y_t^{e,1} + \frac{exp(\beta * U_{2,t-1})}{\sum_{i=1}^{I} exp(\beta U_{i,t-1})} * y_t^{e,2} \\ &+ \frac{exp(\beta * U_{3,t-1})}{\sum_{i=1}^{I} exp(\beta U_{i,t-1})} * y_t^{e,3} + \frac{exp(\beta * U_{4,t-1})}{\sum_{i=1}^{I} exp(\beta U_{i,t-1})} * y_t^{e,4} \end{aligned} \tag{3.12}$$

The switching mechanism is based on multinomial logit probabilities with learning speed β. If $\beta = 0$, there is a situation of *perfect heterogeneity*, while $\beta = \infty$ implies that agents recognize differences in the heuristics immediately, which corresponds to *perfect homogeneity*. Equation (3.12) is estimated using NLS in order to identify parameter β. In line with the simulations conducted by Assenza et al. (2014), the inital value of β is set to 0.4. The results are shown in table 7. The coefficient is positive, which means that agents react to differences in forecasting errors. However, it is not statistically significant.

The estimation can be repeated for Eurozone data. This time the coefficient comes close to being statistically significant. The magnitude of the coefficients is comparable to the estimates by Cornea/Hommes/Massaro (2012). One reason for this weak evidence might lie in the fact that output gap variations are not only induced by expectations, but by other variables as well. Nevertheless, the R^2 is around 90% in both cases. Figure 5 shows the implied time paths of the four fractions based on Eurozone data. There is a lot of time variation due to forecasting errors, although we cannot strictly rule out perfect heterogeneity as well, as the fractions are roughly one quarter on average. During the Great recession, adaptive expectations seem to increase, while trend-followers collapse. This is especially true for strong trend-followers. If we summarize all rules containing a trend-component and compare the share of trend-followers to the share of non-trend-followers, this story is confirmed (see Figure 6). Trend-followers have always a larger share than adaptive expectations, indicating that only a minority of agents (around one quarter) is willing to learn from past mistakes. Adaptive expectations fluctuate between a size close to zero and one half. As the economy gets better, the share of trend-followers increases and the share of adaptive expectations drops, as can be seen in the dotcom bubble before 2000. The eurocrisis that occured after the financial crisis also led to an increase of adaptive expectations, especially in 2015, when the Greek crisis was severe. Figure 6 also illustrates that the correlation of group shares with the business cycle is far from being perfect. Somewhat surprisingly, the share of trend-followers is countercyclical with a correlation coefficient of -0.0273, while adaptive expectations are procyclical at +0.0273. The reason for this might lie in the lag structure, as weights of the heuristics are determined by past forecasting errors, which is consistent with the fact that past values of trend-followers are procyclical and past values of adaptive expectations are countercyclical. Moreover, the autocorrelation for both time series is 0.4422, which shows some inertia in the updating process.

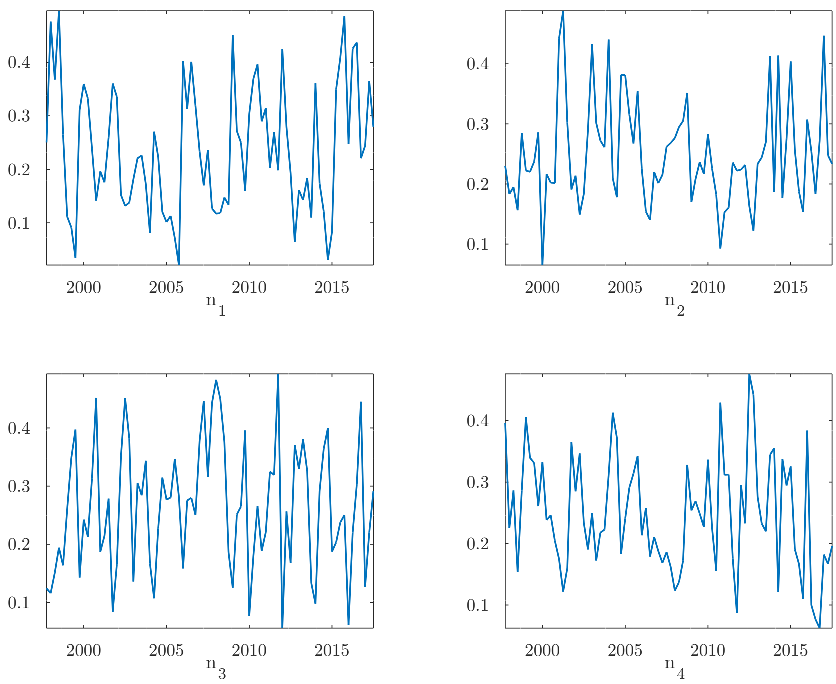

Figure 5: Implied shares, own figure

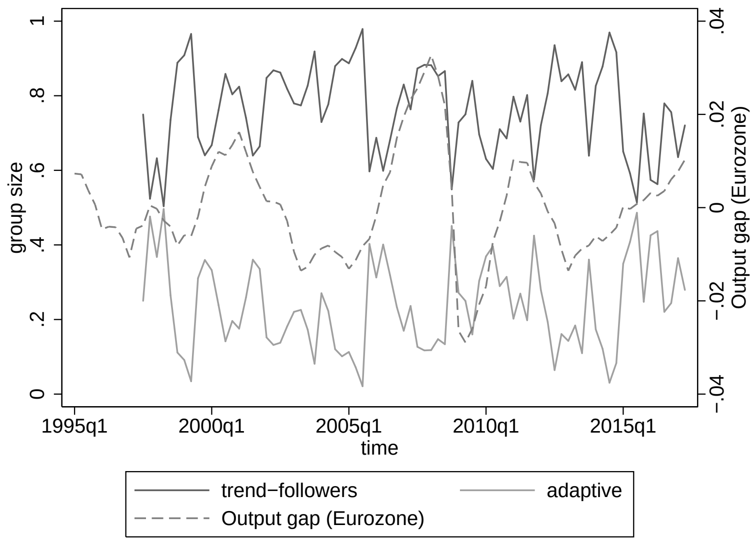

Figure 6: Comparison of trend-followers and adaptive expectations

4 Conclusion

The present paper has analyzed the empirical relationship between expectations and volatility. Empirical expectations data show a strong comovement between forecasts and economic variables. The causality runs in both directions, i.e. expectations influence economic variables, but they are also influenced by past observations of economic variables. A structural estimation shows large fractions of trend-following heuristics. Assuming time-varying fractions leads to results that are closer to perfect heterogeneity than to perfect homogeneity. As many studies have shown a negative link between volatility and growth, the bounded rationality of heterogeneous expectations might also matter in the long run. There are many directions for future research. First, the use of heterogeneous expectation heuristics can be related to the level of educational attainment. While evolutionary switching still assumes a form of homogeneity, as agents can use all strategies and switch between complex and less complex heuristics, a more realistic approach would consider the relation between expectations and cognitive abilities, which are influenced by education. Second, these macroeconometric findings can be more closely linked with micro evidence of consumption and investment decisions. Finally, another possibility is to look at individual countries because of the high structural heterogeneity of the European economy that also affects business cycles and seems to be higher than in the US.

References

Anufriev, M./Hommes, C. (2012): Evolutionary Selection of Individual Expectations and Aggregate Outcomes in Asset Pricing Experiments. *American Economic Journal: Microeconomics* 4 (4), pp. 35–64.

Assenza, T./Heemeijer, P./Hommes, C./Massaro, D. (2014): Managing Self-organization of Expectations through Monetary Policy: a Macro Experiment. *Working Paper* .

Boswijk, H. P./Hommes, C. H./Manzan, S. (2007): Behavioral heterogeneity in stock prices. *Journal of Economic Dynamics & Control* 31, pp. 1938–1970.

Clarida, R./Galí, J./Gertler, M. (2000): Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory. *Quarterly Journal of Economics* 115 (1), pp. 147–180.

Cole, S. J./Milani, F. (2016): The Misspecification of Expectations in New Keynesian Models: A DSGE-VAR Approach. *CESifo Working Paper* 6099.

Cornea, A./Hommes, C./Massaro, D. (2012): Behavioral Heterogeneity in U.S. Inflation Dynamics. *Working Paper* .

Curtin, R. (2007): Consumer Sentiment Surveys: Worldwide Review and Assessment. *Journal of Business Cycle Measurement and Analysis* pp. 1–37.

European Commission (2017): The Joint Harmonised EU Programme of Business and Consumer Surveys. *User guide* .

Garnitz, J./Wollmershäuser, T. (2017): Neuerungen im Ifo World Economic Survey. *ifo Schnelldienst* 70 (1), pp. 24–28.

Hommes, C. (2013): *Behavioral Rationality and Heterogeneous Expectations in Complex Economic Systems*. Cambridge: Cambridge University Press.

Pfajfar, D./Žakelj, B. (2014): Experimental evidence on inflation expectation formation. *Journal of Economic Dynamics & Control* 44, pp. 147–168.